**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:-**The work begin after 10 min, so the average time increase from 45min to 55min**.

**for normal distribution :-**

**z = (X-μ)/б**

**= (60-55)/8**

**= 0.625**

**As we want to find the probability of service manager cannot meet his commitment.**

**1-pnorm(0.625)**

**=0.2659**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans. Mean = 38**

**SD = 6**

1. **Z score = (Value - Mean)/SD**

**Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %**

**People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400**

**Z score for 38  = (38 - 38)/6 = 0 => 50%**

**Hence People between 38 & 44  age**

**= 84.13 - 50 = 34.13 % ≈  137 out of 400**

**Hence More employees at the processing center are older than 44 than between 38 and 44. is** F**ALSE**

1. **Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400**

**Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - TRUE**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans) Here X1 and X2 are two independent random variables then**

**X1+X2 ~N(µ+µ,σ2+σ2)andX1-X2~N(µ-µ,σ 2σ2)**

**2X1~N(2µ,2σ2)**

**2X1-(X1+X2) = N(2µ,2σ2)-N(µ+µ,σ2+σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:-**

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

**Given:  p(a<x<b) = 0.99 ,m ean =100,standardDeviation = 20**

**To Find:**

**Identify symmetric values for the standard normal distribution such that the area enclosed is .99**

**From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values**

**Using Python**

**Z value is given as stats.norm.ppf(pvalue)**

**Z value at 0.5th percentile is given as**

**Z(0.5) = stats.norm.ppf(0.005)= -2.576**

**Z value at 99.5 percentile is given as**

**Z(99.5) = stats.norm.ppf(0.995) = 2.576**

**Z = (x - 100)/20 = > x = 20z+100**

**a = -(20\*2.576) + 100= 48.5**

**b = (20\*2.576)+100= 151.5**

**Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-**import numpy as np**

**from scipy import stats**

**from scipy.stats import norm**

**Mean = 5+7**

**print('Mean Profit is Rs', Mean\*45,'Million')**

**SD = np.sqrt((9)+(16))**

**print('Standard Deviation is Rs', SD\*45, 'Million')**

**print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')**

**X= 540+(-1.645)\*(225)**

**print('5th percentile of profit (in Million Rupees) is',np.round(X,))**

**print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')**

**X= 540+(-1.645)\*(225)**

**print('5th percentile of profit (in Million Rupees) is',np.round(X,))**

**stats.norm.cdf(0,5,3)**

**stats.norm.cdf(0,7,4)**